

This is a "Snapshot" of a User's work in "Introductions," the first of 15 paths in the MemArith program:

In the Sequences below, the program provided the first three problems in each column. The User typed the subsequent problems and answers in each column, following the pattern established in the first three problems. After the User typed several problems correctly, the program completed that column, indicating that the User knew and understood the pattern.

Each wrong key the User typed appeared in red for a few seconds, then disappeared, as the red "=" below is about to disappear, to allow the User to try again. The Feedback at the bottom of the screen shows this is the first Wrong Key the User typed in the current column.

Each Right Key the User typed appears in bright green.

Two Slow Keys (subtraction signs) appear in pink. The program provided these figures when the User was slow to respond.

Easy problems at the start limit a beginner's focus to gaining keyboard accuracy and understanding how the program works. These first two mysteries are easily mastered if the math is not also mysterious.

Sequence No. 103

Usable Keys: (Nn)ext.Sequence (Ee)xplore <Esc>

$0 + 0 = 0$

$1 + 0 = 1$

$2 + 0 = 2$

$3 + 0 = 3$

$4 + 0 = 4$

$5 + 0 = 5$

$6 + 0 = 6$

$7 + 0 = 7$

$8 + 0 = 8$

$9 + 0 = 9$

$10 + 0 = 10$

$11 + 0 = 11$

$12 + 0 = 12$

$13 + 0 = 13$

$14 + 0 = 14$

$15 + 0 = 15$

$0 + 0 = 0$

$0 + 1 = 1$

$0 + 2 = 2$

$0 + 3 = 3$

$0 + 4 = 4$

$0 + 5 = 5$

$0 + 6 = 6$

$0 + 7 = 7$

$0 + 8 = 8$

$0 + 9 = 9$

$0 + 10 = 10$

$0 + 11 = 11$

$0 + 12 = 12$

$0 + 13 = 13$

$0 + 14 = 14$

$0 + 15 = 15$

$0 - 0 = 0$

$1 - 0 = 1$

$2 - 0 = 2$

$3 - 0 = 3$

$4 - 0 = 4$

$5 =$

Feedback: RightKey 16 WrongKey 1 SlowKey 2 Ave.Sec 0.112

The MemArith program offers 15 separate paths, with 99 sequences (columns) in each path, and 16 related problems per sequence. The 15 paths differ as to learning goals and level of difficulty.

When the User exceeds the time limit for predicting any next key, the program provides the next character in pink. The User can control the time limit. Typing <F> (faster) gives 10% less time and <S> (slower) gives 10% more time to predict each next key/character.

All Slow Keys, Wrong Keys and Right Keys in the current column are counted as the User predicts each next key by typing it.

In the “snapshot” moment you see below, sequence 104 has just completed itself after the User typed “5+ 0= 5” and “ 6+ 0= 6” perfectly, with every key quick enough. The User now has time to study the pattern and check the results and speed for that sequence (column) at the bottom of the page. When the User is ready, typing “N” starts the next column of problems to predict.

Notice that the Feedback shows 1 Slow Key, but there are no pink numbers in the last sequence. The slow key was a space bar. Spaces are invisible in any color, but they do count! Each blank space is a symbol that means “positive,” just as “-” is a symbol that means “negative.”

Every symbol, every number, and every space has a meaning that is a required part of the problem.

Sequence No. 104 Usable Keys: (Nn)ext.Sequence (Ee)xplore <Esc>

0 + 0 = 0	0 + 0 = 0	0 - 0 = 0	0 - 0 = 0
1 + 0 = 1	0 + 1 = 1	1 - 0 = 1	1 - 1 = 0
2 + 0 = 2	0 + 2 = 2	2 - 0 = 2	2 - 2 = 0
3 + 0 = 3	0 + 3 = 3	3 - 0 = 3	3 - 3 = 0
4 + 0 = 4	0 + 4 = 4	4 - 0 = 4	4 - 4 = 0
5 + 0 = 5	0 + 5 = 5	5 - 0 = 5	5 - 5 = 0
6 + 0 = 6	0 + 6 = 6	6 - 0 = 6	6 - 6 = 0
7 + 0 = 7	0 + 7 = 7	7 - 0 = 7	7 - 7 = 0
8 + 0 = 8	0 + 8 = 8	8 - 0 = 8	8 - 8 = 0
9 + 0 = 9	0 + 9 = 9	9 - 0 = 9	9 - 9 = 0
10 + 0 = 10	0 + 10 = 10	10 - 0 = 10	10 - 10 = 0
11 + 0 = 11	0 + 11 = 11	11 - 0 = 11	11 - 11 = 0
12 + 0 = 12	0 + 12 = 12	12 - 0 = 12	12 - 12 = 0
13 + 0 = 13	0 + 13 = 13	13 - 0 = 13	13 - 13 = 0
14 + 0 = 14	0 + 14 = 14	14 - 0 = 14	14 - 14 = 0
15 + 0 = 15	0 + 15 = 15	15 - 0 = 15	15 - 15 = 0

Feedback: RightKey 31 WrongKey 2 SlowKey 1 Ave.Sec 0.072

This sample screen shows sequences 9 through 12 in the same “Introductions” path used in Snapshots 1 and 2.

At the start of the session, the User has chosen 0 instead of 2 as the “Level of Difficulty,” the number of consecutive perfectly correct problems the User needs to type for the program to complete the sequence.

Choosing a 0 “Level of Difficulty” means no problems of correct input from the User are needed before the **program will complete the column**. Typing <N> (Next Sequence) will print the whole next column at once. Because the **program provides all the input**, no numbers appear in Feedback.

This User is browsing, studying each sequence, and comparing it to other columns on that screen.

Browsing helps the User get ideas to use in future sessions to predict more quickly and accurately. Browsing lets the User study connections between columns, and between lists of numbers in each column. The lists of numbers below may seem very familiar to readers who read down each list.

The User who studies or predicts the problems on this sample screen may learn some multiplication they didn’t plan to learn, before they ever hear of multiplication.

Sequence No. 112 Usable Keys: (Nn)ext.Sequence (Ee)xplore <Esc>

0 + 0 = 0	0 + 0 = 0	0 + 0 = 0	0 + 0 = 0
1 + 2 = 3	3 + 1 = 4	3 - 1 = 2	3 + 2 = 5
2 + 4 = 6	6 + 2 = 8	6 - 2 = 4	6 + 4 = 10
3 + 6 = 9	9 + 3 = 12	9 - 3 = 6	9 + 6 = 15
4 + 8 = 12	12 + 4 = 16	12 - 4 = 8	12 + 8 = 20
5 + 10 = 15	15 + 5 = 20	15 - 5 = 10	15 + 10 = 25
6 + 12 = 18	18 + 6 = 24	18 - 6 = 12	18 + 12 = 30
7 + 14 = 21	21 + 7 = 28	21 - 7 = 14	21 + 14 = 35
8 + 16 = 24	24 + 8 = 32	24 - 8 = 16	24 + 16 = 40
9 + 18 = 27	27 + 9 = 36	27 - 9 = 18	27 + 18 = 45
10 + 20 = 30	30 + 10 = 40	30 - 10 = 20	30 + 20 = 50
11 + 22 = 33	33 + 11 = 44	33 - 11 = 22	33 + 22 = 55
12 + 24 = 36	36 + 12 = 48	36 - 12 = 24	36 + 24 = 60
13 + 26 = 39	39 + 13 = 52	39 - 13 = 26	39 + 26 = 65
14 + 28 = 42	42 + 14 = 56	42 - 14 = 28	42 + 28 = 70
15 + 30 = 45	45 + 15 = 60	45 - 15 = 30	45 + 30 = 75

Feedback: Right Key <> Wrong Key <> Slow Key <> Ave.Sec .<>

On this “snapshot” screen, the User is in “Explore” Mode.

The User provided the **first two problems** in each sequence (column), with the program providing **answers**. Each column’s **third problem and answer** were provided by the program, using the pattern established by the first 2 problems.

Once the first 3 problems are on the screen, Explore Mode works exactly like predicting any sequence of problems. Explore Mode invites learners to open their own investigations.

Usable Keys: 1 2 3 4 5 6 7 8 9 0 + - * = <Sp> <Esc>
Ff Ss Ii Jj Ee

100 - 11 = 89	1000 - 11 = 989	1000 - 111 = 889	1000 - 0 = 1000
100 - 22 = 78	1000 - 22 = 978	1000 - 222 = 778	1000 - 101 = 899
100 - 33 = 67	1000 - 33 = 967	1000 - 333 = 667	1000 - 202 = 798
100 - 44 = 56	1000 - 44 = 956	1000 - 444 = 556	1000 - 303 = 697
100 - 55 = 45	1000 - 55 = 945	1000 - 555 = 445	1000 - 404 = 596
100 - 66 = 34	1000 - 66 = 934	1000 - 666 = 334	1000 - 505 = 5
100 - 77 = 23	1000 - 77 = 923	1000 - 777 = 223	
100 - 88 = 12	1000 - 88 = 912	1000 - 888 = 112	
100 - 99 = 1	1000 - 99 = 901	1000 - 999 = 1	
100 - 110 = -10	1000 - 110 = 890	1000 - 1110 = -110	
100 - 121 = -21	1000 - 121 = 879	1000 - 1221 = -221	
100 - 132 = -32	1000 - 132 = 868	1000 - 1332 = -332	
100 - 143 = -43	1000 - 143 = 857	1000 - 1443 = -443	
100 - 154 = -54	1000 - 154 = 846	1000 - 1554 = -554	
100 - 165 = -65	1000 - 165 = 835	1000 - 1665 = -665	
100 - 176 = -76	1000 - 176 = 824	1000 - 1776 = -776	

Feedback: RightKey 38 WrongKey 1 SlowKey 4 Ave.Sec 0.198

In this “snapshot,” the User is on a path called “Shortcuts,” which focuses on strategies for simplifying mental addition and subtraction.

The User first sees digits change independently, and then sees how the accumulation of ones flows over to affect the tens digit, and the accumulation of hundreds does the same to thousands.

Seeing individual digits and overflow from 1 digit to the next are 2 key ideas this path begins with.

A variety of simplifying strategies are gradually introduced one at a time in the “Shortcuts” path.

Sequence No. 308

Usable Keys : 1 2 3 4 5 6 7 8 9 0 + - * = <Sp> <Esc>
Ff Ss Ii Jj Ee

$0 + 0 = 0$

$21 + 104 = 125$

$42 + 208 = 250$

$63 + 312 = 375$

$84 + 416 = 500$

$105 + 520 = 625$

$126 + 624 = 750$

$147 + 728 = 875$

$168 + 832 = 1000$

$189 + 936 = 1125$

$210 + 1040 = 1250$

$231 + 1144 = 1375$

$252 + 1248 = 1500$

$273 + 1352 = 1625$

$294 + 1456 = 1750$

$315 + 1560 = 1875$

$0 + 0 = 0$

$203 + 102 = 305$

$406 + 204 = 610$

$609 + 306 = 915$

$812 + 408 = 1220$

$1015 + 510 = 1525$

$1218 + 612 = 1830$

$1421 + 714 = 2135$

$1624 + 816 = 2440$

$1827 + 918 = 2745$

$2030 + 1020 = 3050$

$2233 + 1122 = 3355$

$2436 + 1224 = 3660$

$2639 + 1326 = 3965$

$2842 + 1428 = 4270$

$3045 + 1530 = 4575$

$0 - 0 = 0$

$305 - 203 = 102$

$610 - 406 = 204$

$915 - 609 = 306$

$1220 - 812 = 408$

$1525 - 1015 = 510$

$1830 - 1218 = 612$

$2135 - 1420$

Feedback: RightKey 67 WrongKey 7 SlowKey 4 Ave.Sec 0.199

This “snapshot” is further along the same “Shortcuts” path as Snapshot 5. By now the User is making a habit of comparing each column to its neighbors, looking for every kind of relationship between all the previous problems and the next problem to predict. Each relationship or pattern invites an explanation as to why the pattern works.

Why does the one digit in each column flip back and forth between the same 2 numerals?

Why does the next digit to the left go up one every other number?

What is the relationship between $2+3=5$ and $7+8=15$ or $1+4=5$ and $6+9=15$?

The use of the same numbers in reverse order (commutative property) between the 2nd and 3rd sequence on the screen invites the User to look, in the future, to other sequences on the same screen for additional clues when predicting. The User gradually sees more and more clues in different columns about each column’s patterns,

While all these clues and relationships are available to notice, the math stays relatively simple, to make the relationships between columns easier to notice, and to build confidence while predicting is new.

The organization of patterns provided is more purposeful than memorization techniques such as flash cards. A focus on the question, “what’s the next key,” intensifies the focus on seeing and predicting patterns. The patterns also invite and reward the habits of looking for patterns and thinking about why they are there.

Sequence No. 313

Usable Keys: 1 2 3 4 5 6 7 8 9 0 + - * = <Sp> <Esc>
Ff Ss Ii Jj Ee

7+ 8= 15	6+ 9= 15	9+ 6= 15
12+ 13= 25	11+ 14= 25	14+ 11= 25
17+ 18= 35	16+ 19= 35	19+ 16= 35
22+ 23= 45	21+ 24= 45	24+ 21= 45
27+ 28= 55	26+ 29= 55	29+ 26= 55
32+ 33= 65	31+ 34= 65	34+ 31= 65
37+ 38= 75	36+ 39= 75	39+ 36= 75
42+ 43= 85	41+ 44= 85	44+ 41= 85
47+ 48= 95	46+ 49= 95	49+ 46= 95
52+ 53= 105	51+ 54= 105	54+ 51= 11
57+ 58= 115	56+ 59= 115	
62+ 63= 125	61+ 64= 125	
67+ 68= 135	66+ 69= 135	
72+ 73= 145	71+ 74= 145	
77+ 78= 155	76+ 79= 155	
82+ 83= 165	81+ 84= 165	

Feedback: RightKey 72 WrongKey 6 SlowKey 4 Ave.Sec 0.108